IIT JEE | MEDICAL | FOUNDATION

Solutions to Mock JEE Advanced-1 | 2024 | Paper-1

Mathematics

ONE OR MORE THAN ONE CHOICE

1.(ABCD) (A) Increasing function from A to B is ${}^{8}C_5 = {}^{8}C_3 = 56$

(B) Non-decreasing function from A to B is ${}^{12}C_5 = 792$

(C) No. of onto function A to A, such that $f(i) \neq i$ is equivalent to derangement of 5 distinct things = 44

(D) Number of onto functions from B to A is $\sum_{r=0}^{4} {}^{5}C_r (5-r)^8 (-1)^r$

2.(BC) Let $y = mx + \frac{1}{m}$ be tangent to parabola $y^2 = 4x$ it will touch the ellipse $\frac{x^2}{4^2} + \frac{y^2}{\left(\sqrt{6}\right)^2} = 1$

if $\frac{1}{m^2} = 16m^2 + 6 \Rightarrow m = \pm\frac{1}{2\sqrt{2}}$

Point of contact for parabola is $\left(\frac{a}{m^2}, \frac{2a}{m}\right)$

So, $A_1(8, 4\sqrt{2})$, $A_4(8, -4\sqrt{2})$

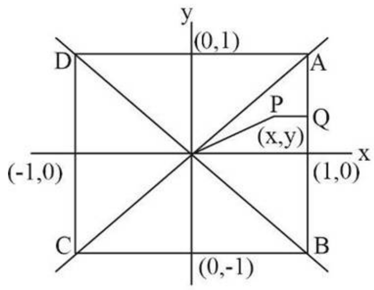
Also, point of contact for ellipse $\left(\mp\frac{a^2 m}{\sqrt{a^2m^2 + b^2}}, \pm\frac{b^2}{\sqrt{a^2m^2 + b^2}}\right)$

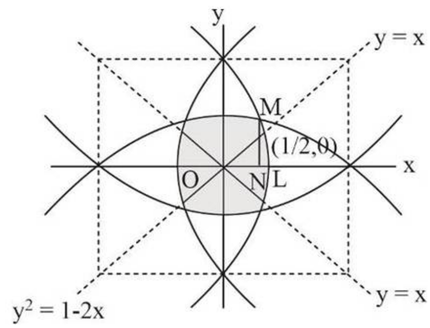
$A_2\left(-2, \frac{3}{\sqrt{2}}\right)$ & $A_3\left(-2, \frac{-3}{\sqrt{2}}\right)$

Then area of trapezium $A_1A_2A_3A_4$ is $55\sqrt{2}$ sq. units

Clearly the tangent T_1 & T_2 meet x-axis at (–8, 0)

3.(AD)





S is the region where $OP \le PQ \;\Rightarrow\; \sqrt{x^2+y^2} \le 1-x$

$$\Rightarrow\; y^2 \le 1-2x$$

Now when the curves $y^2 = 1-2x$ and $y = x$ intersect each other then

$x^2 = 1-2x \;\Rightarrow\; x = \sqrt{2}-1,\, -\sqrt{2}-1$

Hence the intersection points in the first quadrant is $\left(\sqrt{2}-1,\, \sqrt{2}-1\right)$

The required area = 8 (area of curvilinear ΔOLM)

$$= 8\left[\frac{1}{2}\left(\sqrt{2}-1\right)\left(\sqrt{2}-1\right) + \int_{\sqrt{2}-1}^{1/2} \sqrt{1-2x}\, dx\right] = \frac{4}{3}\left(4\sqrt{2}-5\right) = \frac{4}{3}\left(a\sqrt{2}-b\right)$$

So, $a = 4$ & $b = 5$

SINGLE CHOICE

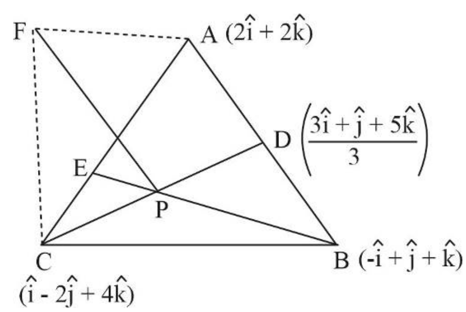
4.(A) $\dfrac{2f(x)\cdot f'(x)}{\sqrt{1-(f(x))^4}} - 2x \ge 0 \;\Rightarrow\; \dfrac{d}{dx}(\sin^{-1}(f(x))^2 - x^2) \ge 0$

Let $g(x) = \sin^{-1}((f(x)^2) - x^2$ is a non-decreasing function

$$\lim_{x\to k_1^+} g(x) \le \lim_{x\to k_2^-} g(x) \;\Rightarrow\; \frac{\pi}{2} - k_1^2 \le \frac{\pi}{6} - k_2^2 \;\Rightarrow\; k_1^2 - k_2^2 \ge \frac{\pi}{3} \;\Rightarrow\; \left[k_1^2 - k_2^2\right] \ge 1$$

5.(B) $\overrightarrow{CD} : \vec{r} = (\hat{i} - 2\hat{j} + 4\hat{k}) + \dfrac{\lambda}{3}(7\hat{j} - 7\hat{k})$

$BE : \vec{r} = (-\hat{i} + \hat{j} + \hat{k}) + \dfrac{\mu}{3}(7\hat{i} - 7\hat{j} + 7\hat{k})$



$P \equiv (\hat{i} - \hat{j} + 3\hat{k})$

$\overrightarrow{AB} \times \overrightarrow{AC} = 7\hat{j} + 7\hat{k}$

$\left|\overrightarrow{PF}\right| = \sqrt{2}$ units

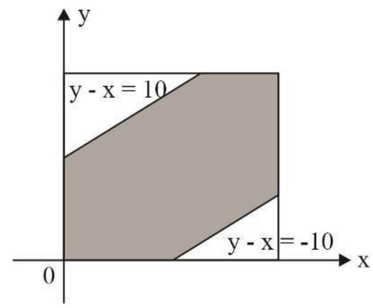
$\overrightarrow{PF} = \sqrt{2}\left(\dfrac{7\hat{j} + 7\hat{k}}{\sqrt{49+49}}\right) = \hat{j} + \hat{k} =$ p.v. of F – p.v. of P

p.v. of $F = \hat{i} + 4\hat{k}$

6.(D) The sample space is

$S=\{(x,y)\,;\,0\le x,y\le 30\}$, $A=\{(x,y):|x-y|\le 10\}$

Hence the probability $=\dfrac{30\times 30-\frac{1}{2}\times 20\times 20-\frac{1}{2}\times 20\times 20}{30\times 30}=5/9$



7.(C) Let $A(\alpha,\beta)$, the equation of the normal to the parabola at $(at^2,2at)$ is $y+xt=2at+at^3$

As it passes through (α,β) so $at^3+(2a-\alpha)t-\beta=0$ have three roots t_1,t_2,t_3

$\Rightarrow\ t_1+t_2+t_3=0 \qquad \ldots$(i)

Now $P(at_1^2,\,2at_1)$, $Q(at_2^2,\,2at_2)$ and $R(at_3^2,\,2at_3)$

Equation of $C_1\equiv(x-\alpha)(x-at_1^2)+(y-\beta)(y-2at_1)=0$

$C_2\equiv(x-\alpha)(x-at_2^2)+(y-\beta)(y-2at_2)=0$

Common chard $\equiv(t_1+t_2)x+2y-\alpha(t_1+t_2)-2\beta=0$

From (i) $\qquad \therefore\ m_1=-\left(\dfrac{t_1+t_2}{2}\right)=\dfrac{t_3}{2}$

Tangent to the parabola at R $\qquad t_3y=x+at_3^2\ \Rightarrow\ m_2=\dfrac{1}{t_3}$ $\qquad$ So, $m_1\times m_2=\dfrac{1}{2}$

MATRIX MATCH TYPE

8.(D) $D=(\lambda-2)(\mu-3)$; $D_1=(\lambda-2)(4\mu-15)$; $D_2=0$; $D_3=(\lambda-2)$

$x=\dfrac{D_1}{D},\ y=\dfrac{D_2}{D},\ z=\dfrac{D_3}{D}$

For no solution $\Rightarrow\ \lambda\ne 2,\ \mu=3$; For unique solution $\Rightarrow\ \lambda\ne 2,\ \mu\ne 3$

For infinite many solution $\Rightarrow\ \lambda=2,\ \mu\in R$; Exactly three solution is not possible

9.(A)

x_i	2	5	6	8	10	12
f_i	2	8	12	4	8	6
c_f	2	10	22	26	34	40

N = 40 (even)

$\dfrac{N}{2}=20$

Median (M) will be 6

Mean $(\bar{x})=\dfrac{\Sigma x_i f_i}{\Sigma f_i}=\dfrac{2\times 2+5\times 8+6\times 12+8\times 4+10\times 8+12\times 6}{40}=\dfrac{4+40+72+32+80+72}{40}=7.5$

Mean deviation about mean $=\dfrac{1}{N}\Sigma f_i\left|(x_i-\bar{x})\right|$

$=\dfrac{1}{40}\left(2\times 5.5+8\times 0.5+12\times 1.5+4\times 0.5+8\times 2.5+6\times 4.5\right)=\dfrac{1}{40}(11+4+18+2+20+27)=2.1$

Mean deviation about median $=\dfrac{1}{N}\Sigma f_i\left|x_i-M\right|$

$=\dfrac{1}{40}(2\times 4+8\times 1+12\times 0+4\times 2+8\times 4+6\times 6)=\dfrac{1}{40}(8+8+8+32+36)=2.3$

10.(B) $|z_1| = 1$ & $|z_2| = 2$

I. $|z_1 + z_2| \le |z_1| + |z_2| \le 3$ II. $|2z_1 - z_2| \ge |2|z_1| - |z_2|| \ge 0$

III. $|2z_1 + z_2| \le 2|z_1| + |z_2| \le 4$ IV. $|3z_1 - 2z_2| \ge |3|z_1| - 2|z_2|| \ge 1$

11.(C) Any point on L_1 is $(2\lambda+1, -\lambda, \lambda-3)$ and that on L_2 is $(\mu+4, \mu-3, 2\mu-3)$.

For point of intersection of L_1 and L_2,

$2\lambda+1 = \mu+4, -\lambda = \mu-3, \lambda-3 = 2\mu-3 \Rightarrow \lambda = 2, \mu = 1$

Intersection point of L_1 & L_2 is (5, -2,-1)

$ax+by+cz=d$ is perpendicular to P_1 & P_2

$\therefore\ 7a+b+2c=0$ and $3a+5b-6c=0$

$\Rightarrow\ 7a+b+2c=0$ and $3a+5b-6c=0$

$\Rightarrow\ \dfrac{a}{-16} = \dfrac{b}{48} = \dfrac{c}{32} \Rightarrow \dfrac{a}{1} = \dfrac{b}{-3} = \dfrac{c}{-2}$

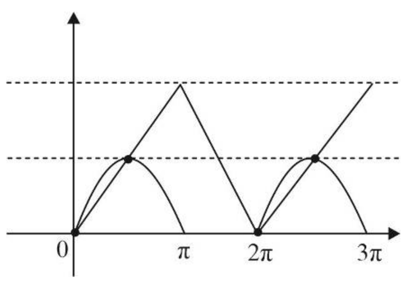
Equation of plane is $x-3y-2z=d$

as it passes through $(5,-2,-1)$

$5+6+2=d \Rightarrow d=13$ $\therefore\ a=1, b=-3, c=-2, d=13$

NON-NEGATIVE INTEGER TYPE

1.(3) Total 3 solutions by graph



2.(0) $f(x) = (\ln x)^2 - \displaystyle\int_1^e \frac{f(t)}{t}\,dt$

Let $\displaystyle\int_1^e \frac{f(t)}{t}\,dt = K$

$f(x) = (\ln x)^2 - K$

where, $K = \displaystyle\int_1^e \frac{(\ln t)^2 - K}{t}\,dt$; Let $\ln t = z \Rightarrow \dfrac{dt}{t} = dz$

$= \displaystyle\int_0^1 \left(z^2 - K\right) dz = \frac{1}{3} - K$

$K = \dfrac{1}{6}$

$f(x)=(\ln x)^2-\frac{1}{6}$

$f(e)=\frac{5}{6}$

$f'(x)=\frac{2\ln x}{x}>0\ \forall x>1$

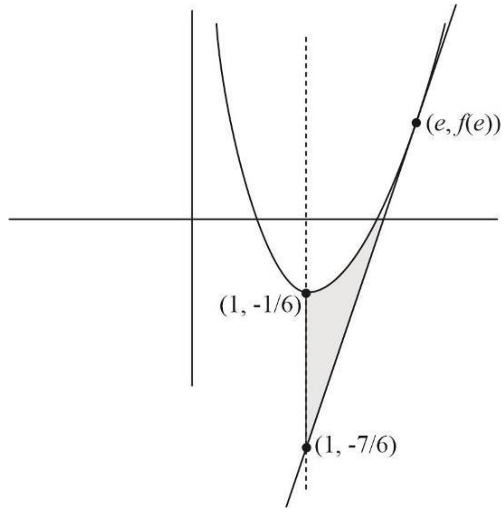
Equation of tangent at $(e,\ f(e))$

$y=\frac{2}{e}x-\frac{7}{6}$

Area bounded

$=\int_1^e\left(f(x)-\left(\frac{2}{e}x-\frac{7}{6}\right)\right)dx$

$A=e+\frac{1}{e}-3$ [A] = 0



3.(37) $S=17\times1+17\times11+17\times111+....17\times111.....1$

Where last term has $n+1$ one's, then

$S=\frac{17}{9}(9+99+999+9999+.....99.....9)=\frac{17}{9}\left[(10-1)+(10^2-1)+(10^3-1)+.....+(10^{n+1}-1)\right]$

$=\frac{17}{9}\left[(10+10^2+10^3+....10^{n+1})-(n+1)\right]=\frac{17}{9}\left[\frac{10^{n+2}-10}{9}-(n+1)\right]=\frac{17}{81}(10^{n+2}-9n-19)$

$=\frac{17}{81}(10^{n+2}-9n-19)$

$a=b=9\ \&\ c=19$

4.(8) Let $z=x+iy$

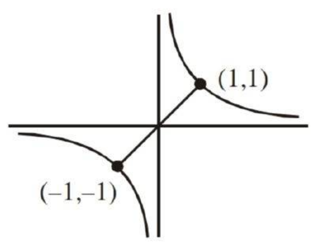
$z^4-|z|^4=4iz^2$

$\Rightarrow\ z^4-(z\bar{z})^2=4iz^2$

$\Rightarrow\ z=0$ or $z^2-(\bar{z})^2=4i$

$\Rightarrow\ 4ixy=4i \quad \Rightarrow\ xy=1$

$|z_1-z_2|^2_{\min}=8$

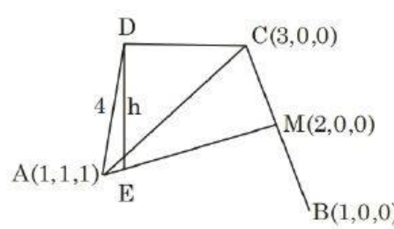


5.(6) $v=\frac{2\sqrt{2}}{3}$

$\Rightarrow\ \frac{1}{3}\cdot\frac{1}{2}\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 0 & 1 & 1\\ 2 & -1 & -1\end{vmatrix}h=\frac{2\sqrt{2}}{3}\ \Rightarrow\ h=2$

Also, Δ ABC is right angled

Now let E divides AM in the ratio $\lambda:1$



$E=\left(\frac{2\lambda+1}{\lambda+1},\frac{1}{\lambda+1},\frac{1}{\lambda+1}\right)$ & $AE^2+DE^2=AD^2$

$\Rightarrow \left(\frac{2\lambda+1}{\lambda+1}-1\right)^2+\left(1-\frac{1}{\lambda+1}\right)^2+\left(1-\frac{1}{\lambda+1}\right)^2+4=16 \qquad \Rightarrow$ Two values of λ

$\frac{3\lambda^2}{(\lambda+1)^2}=12$

$\lambda^2=4(\lambda+1)^2,\ \lambda=-2,\frac{-2}{3}$

$\therefore\ E=(3,-1,-1)$

$EB=\sqrt{4+1+1}=\sqrt{6}$

6.(7) $f(x)=(1+x)^{20}\left[1+\frac{x}{1+x}+\left(\frac{x}{1+x}\right)^2+.....+\left(\frac{x}{1+x}\right)^{18}\right]$

$(1+x)^{21}\left[1-\left(\frac{x}{1+x}\right)^{19}\right]=(1+x)^{21}-(1+x)^2x^{19}$

Coefficient of x^{18} in $f(x)=$ Coefficient of x^{18} in $(1+x)^{21}={}^{21}C_{18}=1330$

Physics

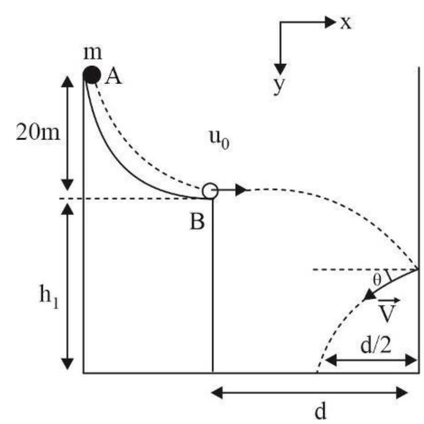
ONE OR MORE THAN ONE CHOICE

1.(ABD) $u_0=\sqrt{2gh}=\sqrt{2\times10\times20}=20\,m/s$

Given, $d=100\,m$

Time taken for ball to reach wall from B

$t=\frac{d}{u_0}=\frac{100}{20}=5\,\text{sec}$



Ball before collision

$\vec{V}_1=u_0(\hat{x})+(gt)\hat{y}$

$\vec{V}_1=20(\hat{x})+50\,\hat{y}m/s$

After collision

$\vec{V}=(-eu_0)\hat{x}+50\hat{y}m/s=-10\hat{x}+50\hat{y}$

$\tan\theta=\left(\frac{50}{10}\right)=5$

$\theta=\tan^{-1}(5)$

Now, time taken by ball to hit ground

After collision $=\frac{\left(\frac{d}{2}\right)}{(eu_0)}=\frac{50}{10}=5\,\text{sec}$

$h_1=\frac{1}{2}g(5+5)^2=\frac{10}{2}\times100=500\,m$

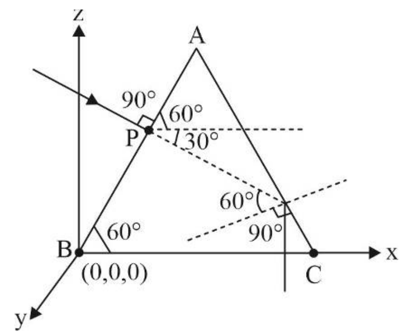
2.(AB) $\sqrt{3}z + x = 10$

$x = 10 - \sqrt{3}z$

Angle of incident light with x-axis 150°

Thus light will pass on its original path

Angle of incidence of surface AC as shown in figure = 60°



For $\mu = \dfrac{2}{\sqrt{3}}$

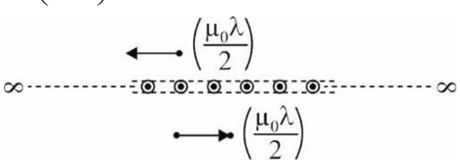
$\sin\theta_C = \dfrac{1}{\mu} \Rightarrow \theta_C = 60°$

i.e., grazing emergence

For $\mu = \dfrac{3}{2} \Rightarrow \theta_C = \sin^{-1}\left(\dfrac{2}{3}\right)$

$\theta_C \approx 41°$

3.(AB) For infinite plate magnetic field at point does not depend on distance from plate to the point and equal to $\left(\dfrac{\mu_0\lambda}{2}\right)$



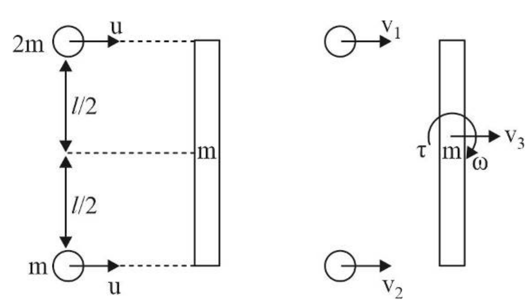
Magnet field at point P = O

Magnet filed at point Q = $\mu_0\lambda$ (toward right)

SINGLE CHOICE

4.(C) Apply COAM with respect to centre of rod

$$\left(2mu\cdot\frac{l}{2} - mu\frac{l}{2}\right) = \left((2mv_1)-(mv_2)\right)\frac{l}{2} + \frac{ml^2}{12}\omega$$



$$\Rightarrow \frac{u}{2} = v_1 - \frac{v_2}{2} + \frac{\omega l}{12} \Rightarrow u = 2v_1 - v_2 + \frac{\omega l}{6}$$

$$\Rightarrow 2v_1 - v_2 + \omega = 6 \qquad \text{(i)}$$

Apply COLM

$$mu + 2mu = mv_2 + 2mv_1 + mv_3$$

$$\Rightarrow 2v_1 + v_2 + v_3 = 3u \qquad \text{(ii)}$$

Collision is elastic

$$v_3 + \frac{\omega l}{2} - v_1 = u$$

$$-v_1 + v_3 + 3\omega = 6 \qquad \text{(iii)}$$

$$v_3 - \frac{\omega l}{2} - v_2 = u$$

$v_3 - 3\omega - v_2 = 6$ (iv)

From eq (ii) – (i)

$2v_2 + v_3 - \omega = 12$ (v)

And from eq. (ii) and (iii)

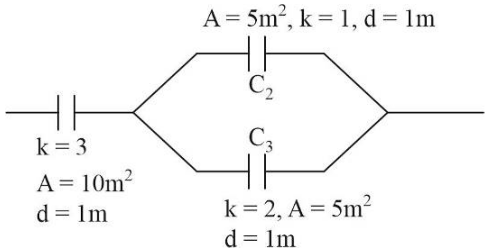
$v_2 + 3v_3 + 6\omega = 30$ (vi)

Solving eq. (iv), (v) and (vi)

$$v_2 = \frac{66}{37} m/s \ ; \ v_3 = \frac{324}{37} m/s \ ; \ \omega = \frac{12}{37} rad/s$$

5.(A) Finally system behaves as shown

Note that as the middle plate is conducting, so C_2 and C_3 are in parallel.



$$C_1 = \frac{kA\varepsilon_0}{d} = 30\varepsilon_0 \ ; \ C_2 = 5\varepsilon_0 \ \& \ C_3 = 10\varepsilon_0$$

$$\left(Cnet = \frac{(15\varepsilon_0)(30\varepsilon_0)}{45\varepsilon_0} \right) = 10\varepsilon_0 = 8.85 \times 10^{-11} F$$

6.(C) $PT^{-B} =$ constant (i)

For any polytropic process $PV^N = Const$

$$C = C_V + \frac{R}{1-N} = \left(\frac{fR}{2} + \frac{R}{1-N} \right) = \frac{5R}{2} + \frac{R}{1-N} \quad \text{(ii)}$$

$PV^N = Const$	$Comparing\ equ\ (i)\ \&\ (ii)$
$P \cdot \left(\frac{nRT}{P} \right)^N = Const$	$-B = \frac{N}{1-N}$
	$\Rightarrow \ NB - B = N$
$PT^{\frac{N}{1-N}} = Const\ (iii)$	$\Rightarrow \ N = \frac{B}{B-1} \quad (iv)$

From equation (ii) & (iv)

$$C = \frac{5R}{2} + \frac{R}{1 - \frac{B}{B-1}} = R\left[\frac{5}{2} + \frac{B-1}{-1} \right] = R\left[\frac{5}{2} + 1 - B \right]$$

$C < 0$

$\frac{7}{2} - B < 0; \quad B > \frac{7}{2}$

7.(A) $g = \dfrac{GM}{r^2}$ [for $r \geq R$]

$$g = \frac{G}{r^2}\cdot\left(\rho\frac{4}{3}\pi R^3\right)\ ;\ g = \left(\frac{4G\rho\pi}{3}\right)\frac{R^3}{(R+h)^2}\ ;\ g \propto \frac{R^3}{(R+h)^2} \quad \text{........(i)}$$

Given : $R_1 = R$ & $R_2 = 2R$

$(g_1)_{at\ surface} = (g_2)_{at\ height\ h}$

$$\Rightarrow \frac{R^3}{(R+0)^2} = \frac{(2R)^3}{(2R+h)^2} \Rightarrow (2R+h)^2 = 8\cdot R^2$$

$$2R + h = 2\sqrt{2}R\ ;\ m = 2R\left(\sqrt{2}-1\right)$$

MATRIX MATCH TYPE

8.(A) I. In the given spontaneous radioactive decay, the number of proton remain constant and all conservation principles are obeyed

II. In fusion reaction of two hydrogen nuclei a proton is decreased as positron shall be emitted in the reaction. All the three conservation principles are obeyed.

III. In the given fission reaction the number of protons remain constant and all conservation principles are obeyed.

IV. In beta negative decay, a neutron transforms into a proton within the nucleus and the electron is ejected out.

9.(A) I. $\lambda_m T = \text{Const}$

$$\Rightarrow \lambda_m T_1 = \lambda_m T_2 \Rightarrow 4753 \times 6050 = 9506 \times T_2 \Rightarrow T_2 = 3025K$$

II. $14450 \times T = 4753 \times 6050 \Rightarrow T = 1990 \simeq 2000K$

III. Visible region lies in between $4000\,\text{Å}$ to $7000\,\text{Å}$ (Approximately)

$$\frac{4753 \times 6050}{7000} < T < \frac{4753 \times 6050}{4000}$$

$4107K < T < 7189K$ So, T can be 5000 K

IV. $\dfrac{dT}{dt} \propto (T - T_0)$

$$\Rightarrow \frac{10}{10} = x\left(\frac{110}{2} - 25\right) \Rightarrow x = \frac{1}{30} \Rightarrow \left(\frac{T-50}{10}\right) = \frac{1}{30}\left(\frac{50+T}{2} - 25\right)$$

$$\Rightarrow T = 42.85°C = 315.8K$$

10.(C) $\because\ \cos\phi = \frac{R}{z}$

$$\cos 30° = \frac{R}{\sqrt{R^2+25}}$$

$$\frac{3}{4} = \frac{R^2}{R^2+25}$$

$$3R^2 + 75 = 4R^2$$

$$R = \sqrt{75} = 5\sqrt{3}\,\Omega$$

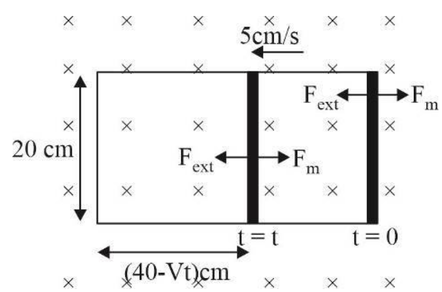
$$z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2+25}\ ;\ z = \sqrt{75+25} = 10\Omega$$

$$\because\ V_R = IR \Rightarrow 2 = I\left(5\sqrt{3}\right)$$

$$I = \frac{2}{5\sqrt{3}}A \quad ; \quad V = Iz = \frac{2}{5\sqrt{3}} \times 10$$

$$V = \frac{4}{\sqrt{3}}V$$

11.(D) $\because\ E_{ind} = \frac{d\phi}{dt} = B\frac{dA}{dt} + A\frac{dB}{dt}$



$B = 2t$ and $A = 20(40 - Vt) \times 10^{-4} m^2$

At $t = 0$

$$E_{ind} = A\frac{dB}{dt} = (20 \times 40) \times 10^{-4}(2)$$

$$E_{ind} = 0.16V$$

$$i = \frac{E_{ind}}{R} = \frac{0.16}{m \times \frac{1\Omega}{m}} = 0.16A$$

At $t = 2s$

$$E_{ind} = 2t \times 20(0-5) \times 10^{-4} + 20(40-5t) \times 10^{-4} \times 2$$

$$E_{ind} = 2 \times 2 \times -100 \times 10^{-4} + 20(40-10) \times 10^{-4} \times 2 = 800 \times 10^{-4} = 0.08V = 80mV$$

$$i = \frac{E_{ind}}{R'} = \frac{0.08}{0.8 \times 1} = 0.1A$$

$$F_{ext} = i\ell B = 0.1 \times 0.2 \times 4 = 0.08N$$

NON-NEGATIVE INTEGER TYPE

1.(2) $\because\ n_2 = 7\ \&\ n_1 = 2$

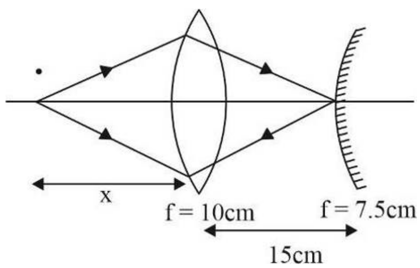
And we know that $\quad eV_0 = E - \phi$

$10eV = E - 2.75eV\ ;\ \ E = 12.75eV \qquad \because\ E = 13.6z^2\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$

$12.75 = 13.6z^2\left[\frac{1}{4} - \frac{1}{49}\right]\ ;\ \ 0.9375 = z^2\left[\frac{49-4}{49\times 4}\right]$

$z^2 = \frac{183.75}{45} = 4.08 \approx 4\ ;\ \Rightarrow\ z = 2$

2.(15) Case-I

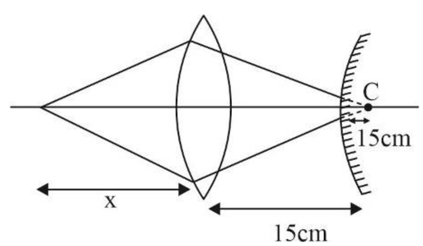


$$\frac{1}{15} - \frac{1}{-x} = \frac{1}{10}$$

$$\frac{1}{x} = \frac{1}{10} - \frac{1}{15} = \frac{3}{30} - \frac{2}{30} = \frac{1}{30}$$

$$x = 30\,cm$$

Case-II $\frac{1}{30} - \frac{1}{-x} = \frac{1}{10}$;



$$\frac{1}{x} = \frac{3}{30} - \frac{1}{30} = \frac{2}{30}$$

$$x = 15\,cm$$

3.(2) Flux linked with a Gaussian sphere of radius r

$= \text{charge enclosed} \times \frac{1}{\varepsilon_0}$

$= \text{volume enclosed} \times \text{charge density} \times \frac{1}{\varepsilon_0}$

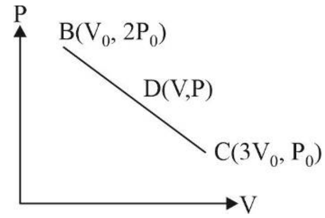
$= \frac{4}{3}\pi r^3 \times \rho \times \frac{1}{\varepsilon_0}$

So, rate of change of flux $= \frac{d\phi_E}{dt} = \frac{d}{dt}\frac{4}{3}\pi\frac{\rho}{\varepsilon_0} \times r^3$

$= \frac{4}{3}\pi\frac{\rho}{\varepsilon_0} \times 3r^2\frac{dr}{dt} = 4\pi r^2\frac{\rho}{\varepsilon_0}\left(\frac{dr}{dt}\right) = 8\pi r^2\frac{\rho}{\varepsilon_0}$

$\therefore \qquad \frac{d\phi_E}{dt} \propto r^2,\ k = 2$

4.(5) $(\Delta U) = \frac{f}{2} \cdot nR(\Delta T)$



During AB process temperature is maximum at B.

And during CA, temperature is maximum at C

$(T_C > T_B)$

$U_C > U_B > U_A$

Lets check in process BC

$B(V_0, 2P_0),\ C(3V_0, P_0)$

Equation of straight line $\frac{P - P_0}{V - 3V_0} = \frac{2P_0 - P_0}{V_0 - 3V_0} = \left(\frac{-P}{2V_0}\right)$

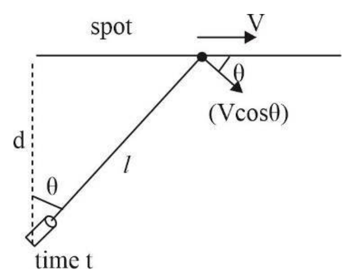
$\Rightarrow (P - P_0)(2V_0) = P_0(3V_0 - V) \quad \Rightarrow P = \frac{(5P_0V_0 - P_0V)}{2V_0} \quad \ldots(i)$

From ideal gas equation $PV = nRT$

For max T, $(PV)_{max}$

$PV = \frac{P_0}{2V_0}(5V_0 - V) \cdot V$ is max at $V = 2.5V_0$; $n = 2.5$; $\quad 2n = 5$

5.(4) $\theta = \omega t$

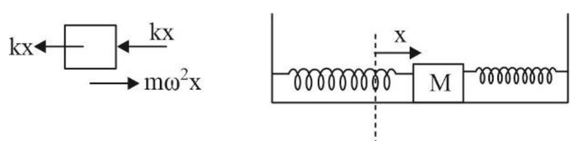


From given diagram tangential component of velocity at time t is $V\cos\theta = \omega l$

$\Rightarrow V\cos\theta = \left(\omega . \frac{d}{\cos\theta}\right)$

$\Rightarrow V = \left(\frac{\omega d}{\cos^2\theta}\right) = \frac{\frac{1}{2} \times 6}{\cos^2\left(\frac{1}{2} \times \frac{\pi}{3}\right)} = \frac{3}{\left(\frac{\sqrt{3}}{2}\right)^2} = 4\, m/s$

6.(8) $F_{net} = m\omega^2 x - 2kx$



$F_{net} = -(2K - m\omega^2)x$

$K_{eff} = 2k - m\omega^2$

For no oscillation $\quad K_{eff} < 0$

$2K - m\omega^2 < 0$; $\omega^2 \geq \frac{2K}{m}$; $|\omega| \geq \sqrt{\frac{2K}{m}}$

Given : K = 16 N/m ; $m = 0.5$ kg ; $|\omega| \geq \sqrt{\frac{2 \times 16}{1/2}} = 8\, rad/\sec$

Chemistry

ONE OR MORE THAN ONE CHOICE

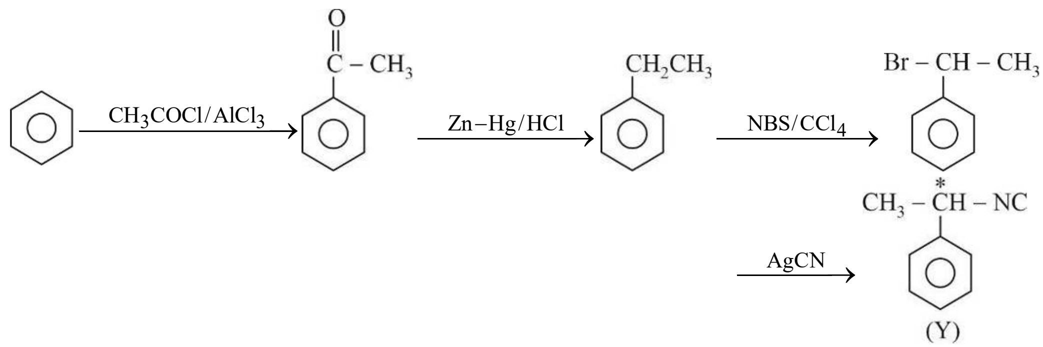
1.(ABD) Chalcocite : Cu_2S and Cinnabar : HgS

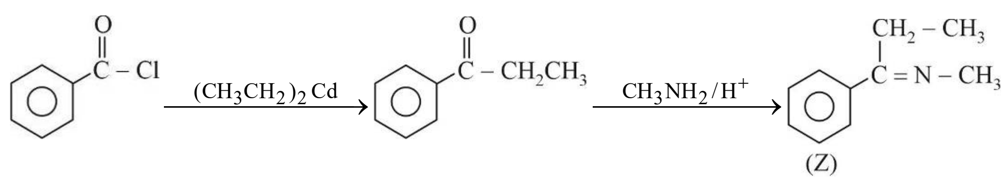
Self-reduction technique is used in the extraction of Cu, Pb and Hg.

2.(ABD)

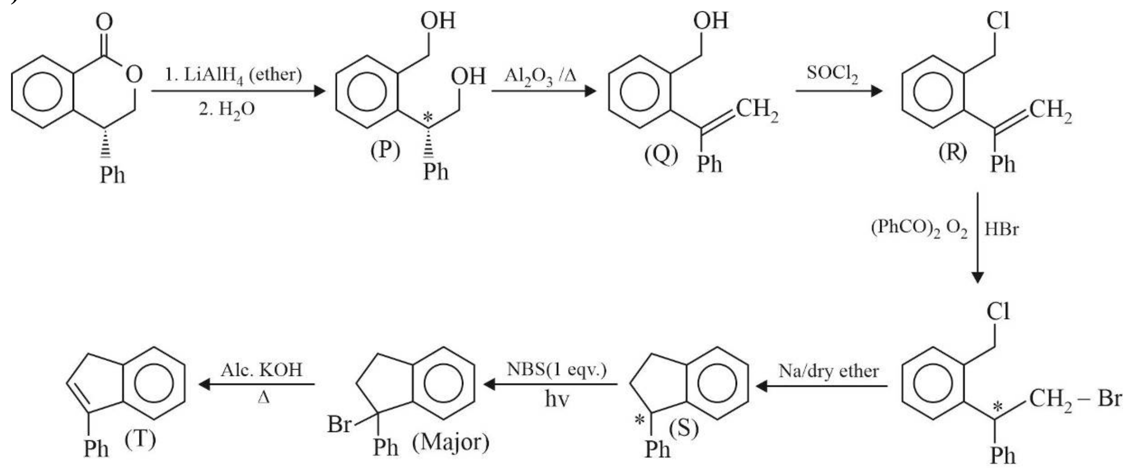
$$CH_3-CH_2-\overset{\overset{\displaystyle O}{\|}}{C}-OCH_3 \xrightarrow{CH_3MgBr} CH_3CH_2-\overset{\overset{\displaystyle O}{\|}}{C}-CH_3 \xrightarrow[\text{(ii) } H_2O]{\text{(i) PhMgBr}} CH_3CH_2-\underset{\underset{\displaystyle Ph}{|}}{\overset{\overset{\displaystyle OH}{|}}{C^*}}-CH_3$$

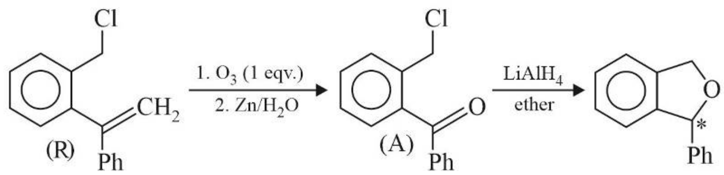
[X]





3.(ACD)





SINGLE CHOICE

4.(B) The reaction is as follows :

$$I_2(g) \xleftarrow{KI} CuSO_4 \xrightarrow{\text{dil. } H_2SO_4} [Cu(H_2O)_4]^{2+} \text{ (Blue colour),}$$

Hence, $X = [Cu(H_2O)_4]^{2+}$, $Y = I_2$

5.(A) $\lambda_m^0 AgCl = (133\times10^{-4} + 426\times10^{-4} - 421\times10^{-4})Sm^2mol^{-1} = (138\times10^{-4})Sm^2mol^{-1}$

For AgCl $\lambda_m = \lambda_m^0$

$\kappa_{AgCl(pure)} = (2.68\times10^{-4} - 0.86\times10^{-4})Sm^{-1} = 1.82\times10^{-4}Sm^{-1}$

$\lambda_m = \dfrac{\kappa}{C}$, here C = solubility

$$\Rightarrow \ c = \frac{1.82\times10^{-4}}{138\times10^{-4}}$$

$c = 0.0132 \text{ mol/m}^3$; $c = 1.32\times10^{-2} \text{ mol/m}^3$; $c = 1.32\times10^{-5} \text{ mol/L}$

$K_{sp}(AgCl) = C^2$; $K_{sp} = 1.74\times10^{-10}$

6.(D) $4.0 = 5.0 + \log\dfrac{C_1}{0.5} \Rightarrow C_1 = 0.05M$

$6.0 = 5.0 + \log\dfrac{C_2}{0.5} \Rightarrow C_2 = 5.0M$

Now, final $pH = 5.0 + \log\dfrac{V\times0.05 + V\times5.0}{V\times0.5 + V\times0.5} = 5.7$

7.(A) Aspirin is non-narcotic drug.

MATRIX MATCH TYPE

8.(B) Theoretical

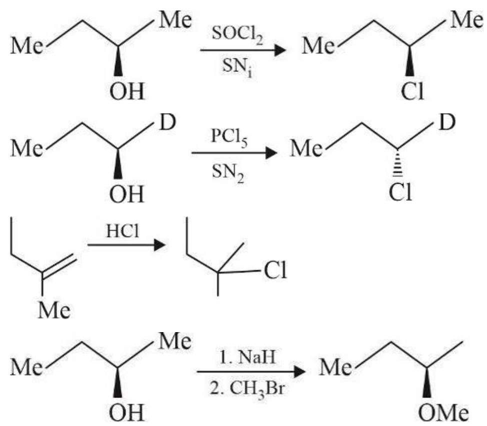
9.(D) $[Ni(CN)_4]^{-2}$: Ni^{+2} ; dsp^2

$[Fe(CN)_6]^{-3}$: Fe^{+3} ; d^2sp^3

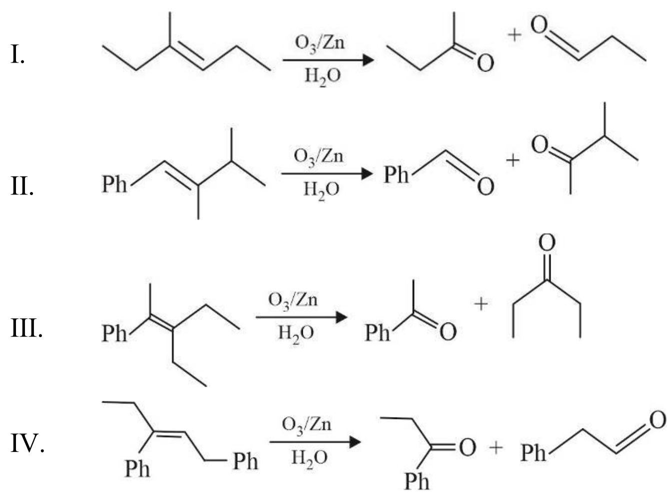
$[NiCl_4]^{-2}$: Ni^{+2} ; sp^3

$[Ni(NH_3)_6]^{+2}$: Ni^{+2} ; sp^3d^2

10.(A)



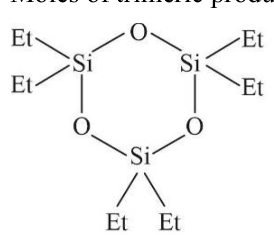
11.(A)



NON-NEGATIVE INTEGER TYPE

1.(153) Moles of diethyldichlorosilane $= \dfrac{471}{157} = 3$

Moles of trimeric product formed $= 1 \times 0.5 = 0.5$



(cyclic trimeric product)

[Molar mass : 306]

2.(3) $P = \frac{ZnRT}{V} = \frac{0.8 \times 1 \times 0.08 \times 300}{0.5}$

P = 38.4 atm

using Vander Waal equation

$$\left(38.4 + \frac{a}{(0.5)^2}\right)(0.5 + 0.02) = (0.08)(300)$$

$38.4 + 4a = 50$

4a = 11.6

a = 2.9

3.(4) $\ln\frac{k_2}{k_1} = \ln\frac{t_1}{t_2} = \frac{E_a}{R}\left(\frac{1}{T_1} - \frac{1}{T_2}\right)$

$\ln\frac{1}{3} = \frac{E_a}{R}\left(\frac{1}{300} - \frac{1}{280}\right)$ …(i)

$\ln\frac{16}{t} = \frac{E_a}{R}\left(\frac{1}{300} - \frac{1}{330}\right)$ …(ii)

From (i) and (ii), t = 4 hrs

4.(560) $q_{ABC} = 600 + 200 = 800\,J$

$w_{AB} = 0$ and $w_{BC} = -\left[8 \times 10^4 \times (5 \times 10^{-3} - 2 \times 10^{-3})\right] \Rightarrow -240\,J$

$\therefore\ \Delta U_{AC} = \Delta U_{ABC} = q_{ABC} + w_{ABC}$

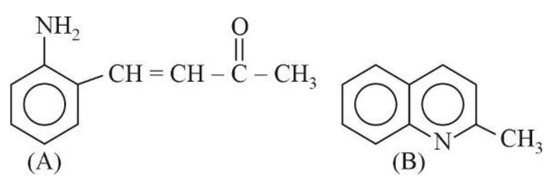
$= 800 + (-240) = 560\,J$

5.(8) $\Delta G_1 = \Delta H - T_1 \cdot \Delta S$ and $\Delta G_2 = \Delta H - T_2 \cdot \Delta S$

$\therefore\ (-\Delta G_2) - (-\Delta G_1) = (T_2 - T_1) \cdot \Delta S = (T_2 - T_1) \times \frac{\Delta H - \Delta G_1}{T_1}$

$= (302 - 298) \times \frac{(-5737) - (-6333)}{298} = 8\ kJ$

6.(2)



D.U.(x) = 6 D.U.(y) = 7

No. of rings(z) = 1 No. of rings(p) = 2

$\frac{y + z}{x - p} = \frac{7 + 1}{6 - 2} = \frac{8}{4} = 2$